SINGULARITIES OF HIGH-FREQUENCY ACOUSTIC PERTURBATION PROPAGATION IN STEAM AND GAS SUSPENSIONS

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A considerable number of papers published at this time is devoted to the problem of linear and slightly nonlinear perturbation propagation. Weak wave propagation was examined in [1-4] and propagation of finite-amplitude perturbations was studied in [5-8]. Brief surveys of papers published earlier can be found in [2, 9, 10].

Results of investigating the dispersion and absorption of weak monochromatic waves with the nonstationary effects of mass, momentum, and energy transfer between the phases taken into account were first published in [4]. In contrast to [4], the case of moderate pressures and moderately small bulk contents of the suspended phase is examined in this paper. A more detailed analysis of the results obtained is presented.

1. Fundamental Equations

Within the framework of a two-velocity and three-temperature model of a gas suspension [11] in a coordinate system with respect to which the unperturbed gas-suspension is at rest ($v_{10} = v_{20} = v_0 = 0$), the linearized plane one-dimensional motion equations can be written in the presence of phase transitions in the form

$$\frac{\partial \rho_1'}{\partial t} + \rho_{10}\frac{\partial v_1'}{\partial x} = -n_0 j, \quad \frac{\partial \rho_2'}{\partial t} + \rho_{20}\frac{\partial v_2'}{\partial x} = n_0 j, \tag{1.1}$$

$$\rho_{10}\frac{\partial v_1'}{\partial t} + \frac{\partial p'}{\partial x} + n_0 f = 0, \quad \rho_{20}\frac{\partial v_2'}{\partial t} = n_0 f,$$

$$\rho_{10}\frac{\partial i_1'}{\partial t} + \rho_{20}\frac{\partial i_2'}{\partial t} = \frac{\partial p'}{\partial t} + n_0 l_0 j,$$

$$\rho_{20}\frac{\partial i_2'}{\partial t} = -n_0 q_{2\sigma} + \alpha_{20}\frac{\partial p'}{\partial t} \quad q_{1\sigma} + q_{2\sigma} = -jl_0,$$

$$\rho_{10} = \alpha_{10}\rho_{10}^0, \quad \rho_{20} = \alpha_{20}\rho_2^0, \quad \alpha_{10} + \alpha_{20} = 1, \quad \alpha_{20} = \frac{4}{3}\pi a_0^3 n_0,$$

where ρ, ρ^0, α, v, i are the mean and true densities, the bulk content, the velocity, and the specific enthalpy; p, pressure; a and n, particle radius and number per unit volume of mixture; l, specific heat of vapor formation; f, force acting from the gas on an individual drop; $q_{j\sigma}$, intensity of the heat transfer between the j-th phase and the surface σ-layer of the drop ($j = 1, 2$); the subscripts 1 and 2 denote parameters of the gaseous and suspended phases; the primes denote parameter perturbations, and the superscript 0 corresponds to the initial unperturbed state.

We limit ourselves to a consideration of the case of homogeneous suspensions when the unperturbed state is homogeneous in the coordinate x, i.e., ρ_{10}, ρ_{20}, n_0, α_{20}, a_0, p_0 = const. We assume that the gaseous phase is a calorically perfect gas, the disperse phase is an ideal incompressible medium whose linearized equations of state have the form

$$\frac{p'}{p_0} = \frac{\rho_1^{0\prime}}{\rho_{10}^0} + \frac{T_1'}{T_0}, \quad i_1' = c_1 T_1'; \quad \rho_2^{0\prime} = 0, \quad i_2' = c_2 T_2' + \frac{p'}{\rho_2^0}, \tag{1.2}$$

where T is the temperature, c_1 and c_2 are the specific heat of the gaseous and condensed phases (at constant pressure). Furthermore, the subscript 0 is omitted where no confusion will result.

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We shall study the solution of the system of linear equations (1.1) and (1.2) which have the form of progressive waves for the perturbations χ':

$$\chi' \sim \exp i(k_* x - \omega t) = \exp(-k_{**}x)\exp i(kx - \omega t) \tag{1.3}$$
$$(k_* = k + ik_{**},\ C_p = \omega/k,\ C_g = d\omega/dk,\ \sigma = 2\pi C_p k_{**}/\omega).$$

Here i is the imaginary unit, k_* is the complex wave number, k_{**} is the linear damping factor, C_p, C_g, and σ are the phase velocity, group velocity, and damping decrement in the wavelength.

2. Phase Interaction Laws

The force acting on an individual spherical particle during its nonstationary motion can be considered as the sum of the quasistationary Stokes viscous friction, the nonstationary Archimedes forces, the apparent masses, and the Bass "hereditary" forces. In the case of the harmonic vibrations (1.3), we have [4]

$$\frac{nf}{\rho_{20}} = \frac{v_1' - v_2'}{\tau_v^*} - \frac{v_1'}{\tau_A^*},\quad \tau_v^* = \tau_v\left[1 + \frac{\sqrt{2}}{2}(1-i)(\omega\tau_{\mu 1})^{1/2} - \frac{1}{9}i\omega\tau_{\mu 1}\right]^{-1}, \tag{2.1}$$
$$\tau_A^* = -\frac{\rho_2^0}{\rho_1^0}\frac{i}{\omega},\quad \tau_v = \frac{2}{9}\frac{\rho_2^0 a^2}{\mu_1} = \frac{2}{9}\frac{\rho_2^0}{\rho_1^0}\tau_{\mu 1},\quad \tau_{\mu 1} = \frac{\rho_1^0 a^2}{\mu_1}\quad (\tau_v \gg \tau_{\mu 1},\ \rho_2^0 \gg \rho_1^0),$$

where $\tau_{\mu 1}$ is the characteristic time of quasistationary velocity distribution buildup in the gas, τ_v is the phase velocity relaxation time for quasistationary gas flow around a particle, τ_v^* is the complex analog of τ_V, and μ_1 is the dynamic viscosity of the gas phase. It is seen that $\tau_v^* \cong \tau_v$ and independent of the vibrations frequency only for sufficiently small frequencies $\omega < 10^{-2}/\tau_{\mu 1}$, when the velocity perturbations penetrate the gas in the characteristic time of the vibrations $\tau = \omega^{-1}$ to the characteristic depth $\delta_{\mu 1} \sim (\mu_1/\rho_1^0\omega)^{1/2}$ that considerably exceeds the drop radius ($\delta_{\mu 1} >> a$). For heterogeneous suspensions $a >> L \sim \mu_1/\rho_1^0 C_1$ (L and C_1 are the molecule mean free path length and the speed of sound in the pure gas), therefore $\tau_{\mu 1} = a^2/LC_1 \gg \tau_C$ ($\tau_C = a/\pi C_1$ is the characteristic time of the vibrations whose wavelength is commensurate with the particle size). An extensive high-frequency band $\tau_{\mu 1}^{-1} < \omega \ll \tau_C^{-1}$ therefore exists for which the phase velocity relaxation time τ_v^* is complex, depends on the vibrations frequency, and can differ radically from the Stokes time τ_V. The condition of acoustic homogeneity of the gas suspension is still not spoiled here, and wave processes therein can be described by the continual equations (1.1).

The Prandtl number of the gas is Pr ~ 1 and the characteristic build-up time of the quasistationary temperature field around the drop in the gas phase is $\tau_{\lambda 1} = a^2/\kappa_1$, where κ_1 is the gas thermal diffusivity coefficient which is of the same order of magnitude as the characteristic build-up time $\tau_{\mu 1}$ of the quaistationary build-up time therein. In this connection the nonstationarity of the temperature fields around particles or drops that results in the dependence of the interphase heat and mass transfer intensities on the frequency ω is manifest at the same vibrations frequencies as the nonstationarity of the velocity fields that results in the distinction between τ_v^* and τ_V. The thermal diffusivity coefficient of the condensed phase κ_2 is ordinarily significantly less than the coefficient κ_1, hence, the characteristic build-up time of the quasistationary homogeneous temperature field within the particle $\tau_{\lambda 2} = a^2/\kappa_2$ is considerably greater than the analogous quantity for a gas ($\tau_{\lambda 2} >> \tau_{\lambda 1}$). And at first glance the nonstationarity of the heat transfer of the drop surface to its fundamental mass which results in a dependence of the heat flux $q_{2\sigma}$ on the frequency, should be manifest for significantly lower vibrations frequencies than the nonstationarity of the drop surface heat transfer to a gas (a more detailed analysis [12] shows that despite $\tau_{\lambda 2} >> \tau_{\lambda 1}$, the nonstationarity of the temperature within the drop is felt in the heat-transfer intensity at considerably higher frequencies than is the nonstationarity in the gas phase).

Dependences analogous to (2.1) for the external $q_{1\sigma}$ and internal $q_{2\sigma}$ thermal fluxes on the frequency ω follow from the solution of the spherically symmetric problems on the heat transfer between a drop and a gas in a monochromatic sound wave and have the form [4, 12]

$$\frac{nq_{1\sigma}}{\alpha_1\rho_{10}^0 c_1}=\frac{T_1'-T_\sigma'}{\tau_{T1}^*},\quad \frac{nq_{2\sigma}}{\alpha_2\rho_2^0 c_2}=\frac{T_2'-T_\sigma'}{\tau_{T2}^*}, \tag{2.2}$$

$$\tau_{T1}^*=\frac{1}{3}\left(\frac{\alpha_1}{\alpha_2}\right)\tau_{\lambda 1}\eta_1(z_1),\quad \tau_{T2}^*=\frac{1}{15}\tau_{\lambda 2}\eta_2(z_2),$$

$$\eta_1(z_1)=\frac{1}{1+z_1},\quad \eta_2(z_2)=\frac{5\left[3z_2-(3+z_2^2)\operatorname{th} z_2\right]}{z_2^2(\operatorname{th} z_2-z_2)},$$

$$z_j=(\omega\tau_{\lambda j})^{1/2}\frac{1-i}{\sqrt{2}},\quad \tau_{\lambda j}=\frac{a^2}{\varkappa_j},\quad \varkappa_j=\frac{\lambda_j}{\rho_j^0 c_j},\quad j=1,2,$$

where τ_{Tj}^* is the complex temperature relaxation time in the j-th phase determined by the characteristic time $\tau_{\lambda j}$ and the frequency ω and λ is the heat conduction coefficient. The time τ_{Tj}^* is close to its quasistationary real value τ_{Tj} for sufficiently small (each phase has its own) frequencies $\tau_{T1}^*\approx\tau_{T1}=(\alpha_1/3\alpha_2)\tau_{\lambda 1}$, $(\omega\tau_{\lambda 1})<10^{-2}(|\eta_1|\sim 1)$, $\tau_{T2}^*\approx\tau_{T2}=(1/15)\tau_{\lambda 2}$, $(\omega\tau_{\lambda 2})<1(|\eta_2|\sim 1)$.

If a phase transformation occurs on the drop surface, then the surface temperature T_σ is determined by its intensity. The Hertz–Knudsen–Langmuir formula [11]

$$\frac{njl}{\alpha_2\rho_1^0 c_1}=\frac{T_S-T_\sigma}{\tau_\sigma},\quad \tau_\sigma=\frac{1-r}{3(\gamma_1-1)}\sqrt{\frac{2\pi}{\gamma_1}}\frac{aC_1^3}{\beta l^2} \tag{2.3}$$

is valid here, where τ_σ is the temperature "relaxation time" at the interphasal boundary that depends on the value of the accommodation (condensation) coefficient β and is independent of the frequency ω, T_S is the saturation temperature which is a known function of the pressure, γ_1 is the pure gas adiabatic index, and $r=\rho_{10}^0/\rho_2^0$.

3. Certain Estimates

Let us execute comparative estimates of the characteristic temperature drops T_1-T_σ, T_2-T_σ, $T_\sigma-T_S$, that are related under acoustic action on a gas suspension. For this we use the equation of heat influx to the interphasal boundary in conformity with

$$\frac{T_1-T_\sigma}{\tau_{\sigma 1}^*}=\frac{T_\sigma-T_2}{\tau_{\sigma 2}^*}+\frac{T_\sigma-T_S}{\tau_\sigma}, \tag{3.1}$$

$$\tau_{\sigma 1}^*=\left(\frac{\alpha_2}{\alpha_1}\right)\tau_{T1}^*=\frac{1}{3}\tau_{\lambda 1}\eta_1(z_1),\ \tau_{\sigma 2}^*=\frac{\rho_1^0 c_1}{\rho_2^0 c_2}\tau_{T2}^*=\frac{\rho_1^0 c_1 a^2}{15\lambda_2}\eta_2(z_2).$$

The left side of this equation is proportional to the thermal flux from the gas to the phase interfacial surface which assures heat expenditure of the phase transition (the second term in the right side) and on a change in the particle temperature (the first term in the right side).

The characteristics times $\tau_{\sigma 1}^*$ and $\tau_{\sigma 2}^*$ used in writing (3.1) differ radically as a rule:

$$\frac{|\tau_{\sigma 2}^*|}{|\tau_{\sigma 1}^*|}=\frac{1}{5}\left(\frac{\lambda_1}{\lambda_2}\right)\frac{|\eta_2|}{|\eta_1|}\ll 1,\ |\eta_2|<|\eta_1|,\ \lambda_1\ll\lambda_2. \tag{3.2}$$

The smallness of the ratio $\tau_\sigma/|\tau_{\sigma 2}^*|$ holds only in definite ranges of the particle size and the vibrations frequency that depend on the physical properties of the phases

$$\frac{\tau_\sigma}{|\tau_{\sigma 2}^*|}=\frac{15}{\beta}\left(\frac{\lambda_2}{\lambda_1}\right)\left(\frac{C_1^2}{l}\right)^2\left(\frac{L}{a}\right)\frac{1}{|\eta_2|}\ll 1, \tag{3.3}$$

where $L\sim\lambda_1/\rho_1^0 c_1 C_1(\gg a)$ is the molecule mean free path in the gas. Thus, for a mixture of steam with water drops in a saturated state with p = 1.0 MPa, when $\lambda_2/\lambda_1\cong 20$, $l/C_1^2\cong 8$, $L\sim 10^{-8}$ we have $\tau_\sigma/|\tau_{\sigma 2}^*|\sim 10^2(L/a)/|\eta_2|$. If the drop characteristic dimension is a $\sim 10^{-5}$ m, then the quantity τ_σ for such a suspension is significantly les than $|\tau_{\sigma 2}^*|$ only for $|\eta_2|\sim 1$, i.e., when the "internal" heat transfer proceeds under quasistationary conditions ($\omega < 1/\tau_{\lambda 2}$).

Taking account of the estimate (3.2), the heat influx equation (3.1) permits making the deduction that the temperature inhomogeneity within the drop (the difference between T_2 and T_σ) being formed under the action of an acoustic field is small compared to the existing temperature inhomogeneity in the gas phase (the difference between T_1 and T_σ). The nonuniformity of the interphasal boundaries (the difference between T_σ and T_S) can be observed only at high vibrations frequencies or small drop sizes when because of the smallness of $|\eta_2|$ or a the quantity $|\tau^*_{\sigma 2}|$ also becomes small and approaches τ_σ (the estimate (3.3)). As a rule, characteristic for acoustic field in gas suspensions is $|T_\sigma - T_S| \ll |T_\sigma - T_2| \ll |T_1 - T_\sigma|$.

If there are no phase transitions, then $q_{1\sigma} = q_{2\sigma}$ and $(T_1 - T_\sigma)/\tau^*_{\sigma 1} = (T_\sigma - T_2)/\tau^*_{\sigma 2}$. Expressing T_σ from this equation, the heat influx equation for the particles can be written in the form

$$\frac{\partial T_2}{\partial t} = \frac{T_1 - T_2}{\tau^*_T}, \tag{3.4}$$

$$\tau^*_T = \frac{\rho^0_2 c_2}{\rho^0_1 c_1}\left(\tau^*_{\sigma 1} + \tau^*_{\sigma 2}\right) \approx \tau^*_{\sigma 1}\frac{\rho^0_2 c_2}{\rho^0_1 c_1} = \tau_T \eta_1(z_1),\ \tau_T = \frac{a^2 \rho^0_2 c_2}{3\lambda_1},$$

where τ_T is the "quasistationary" temperature relaxation time between the phases, and τ^*_T is its complex analog. The estimate (3.2) is used here to simplify the expression for τ^*_T, and in conformity the "internal" nonstationarity of the heat transfer is negligible at any frequencies in the absence of phase transitions.

The results of the analysis performed on the mass, momentum, and heat transfer between the gas suspension phases permit the indication of the characteristic frequency bands of the acoustic fields at which any approximate theories are applicable. In particular the following three fundamental ranges can be isolated:

$$0 \leqslant \omega \ll \left(\tau_v^{-1},\ \tau_T^{-1}\right);$$
$$\left(\tau_v^{-1/2},\ \tau_T^{-1/2}\right) \leqslant \omega^{1/2} \ll \left(\tau_{\mu 1}^{-1/2},\ \tau_{\lambda 1}^{-1/2},\ \tau_{\lambda 2}^{-1/2}\right);$$
$$\left(\tau_{\mu 1}^{-1/2},\ \tau_{\lambda 1}^{-1/2},\ \tau_{\lambda 2}^{-1/2}\right) \leqslant \omega^{1/2} \ll \omega_C\ (\omega_C = \pi C_1/a)$$
$$(\tau_v \sim \tau_T,\ \tau_{\mu 1} \sim \tau_{\lambda 1}).$$

The simplest thermodynamic equilibrium theory is applicable in the first, a nonequilibrium theory using the approximation of quasistationarity of interphasal transfer in the second, and a general nonequilibrium theory taking account of the nonstationary effect of the mass, momentum, and heat transfer between phases. Let us note that despite $\tau_{\lambda 2} >> \tau_{\lambda 1}$, precisely the conditions in the gas phase often determine the upper bound of the frequencies ω at which the quasistationary scheme of interphasal heat transfer can be used [12].

4. Dispersion Dependences

We later use the following dimensionless parameters that characterize the mixture constitution and the physical properties of the phases

$$m = \frac{\rho_2}{\rho_1},\ m^0 = \frac{\rho^0_2}{\rho^0_1},\ r = \frac{\rho^0_1}{\rho^0_2},\ \bar{c}_1 = \frac{c_1}{\gamma_1 R_1} = \frac{1}{\gamma_1 - 1},\ \bar{c}_2 = \frac{c_2}{\gamma_1 R_1},\ \bar{l} = \frac{l}{C_1^2}$$

(R_1 is the gas constant).

We obtain the following dispersion dependence of the wave number k_* on the frequency ω:

$$(C_1 k_*/\omega)^2 = V(\omega)\,\Theta(\omega), \tag{4.1}$$
$$V(\omega) = 1 + m\frac{M^{(1)} - M^{(2)}}{M^{(3)} + mM^{(2)}},\ \Theta(\omega) = 1 + m\frac{\Lambda^{(1)} + (\gamma_1 - 1)\Lambda^{(2)}}{\Lambda^{(3)} + m\Lambda^{(2)}},$$
$$M^{(1)} = (1 - r)/(1 + mr),\ M^{(2)} = r\left[1 - r\left(i\omega\tau^*_v\right)\right],\ M^{(3)} = 1 - \left(i\omega\tau^*_v\right),$$

$$\Lambda^{(1)} = (1-r)\,\bar{l}^{-2}\left[(1-r)\left(\bar{c}_1 + m\bar{c}_2\right) - 2\bar{l} - \right.$$

$$\left. - (1-r)\,m^0\bar{c}_2\left(i\omega\tau^*_{\sigma 1}\right) - \left((1-r)\,\bar{c}_1 - 2\bar{l}\right)\left(i\omega\tau^*_{T2}\right)\right],$$

$$\Lambda^{(2)} = 1 - \left(i\omega\tau^*_{T2}\right) - m^0\left(\bar{c}_2/\bar{c}_1\right)\left(i\omega\tau_\sigma\right),\ \Lambda^{(3)} = -\,m^0\left[\left(i\omega\tau^*_{\sigma 1}\right) + \left(i\omega\tau_\sigma\right) + \right.$$

$$\left. + \left(\omega\tau^*_{\sigma 1}\right)\left(\omega\tau^*_{T2}\right) + m^0\left(\bar{c}_2/\bar{c}_1\right)\left(\omega\tau^*_{\sigma 1}\right)\left(\omega\tau_\sigma\right) + \left(\omega\tau^*_{T2}\right)\left(\omega\tau_\sigma\right)\right]$$

from the conditions for the existence of a nonzero solution of the form (1.3) for the system of linear equations (1.1), (1.2), (2.1)-(2.3). Here $V(\omega)$ and $\theta(\omega)$ are complex functions governed by the particle size and the thermophysical properties of the phases (γ_1, C_1, ℓ, β, μ_1, ρ^0_j, c_j, λ_j, $j = 1, 2$) and which describe the dispersion and dissipative effects because of the relative slip of the phases and the nonequilibrium interphasal heat transfer, respectively. In the absence of particles ($m = 0$), the mentioned effects are not observed, and in this case $V = \theta = 1$. Let us note that the dispersion relationship obtained in [4] and the corresponding ultimately small bulk contents of the suspended phase follow from (4.1) upon passing to the limit $r \to 0$, $\alpha_2 \to 0$.

To estimate the relative contribution of terms responsible for the individual relaxation processes of the interphasal heat and mass transfer in the function $\theta(\omega)$, the complex functions $\Lambda^{(1)}$, $\Lambda^{(2)}$, and $\Lambda^{(3)}$ comprising it are conveniently rewritten so that the ratios of the characteristic relaxation times $\tau^*_{\sigma 1}$, $\tau^*_{\sigma 2}$, and τ_σ that have already been discussed earlier (see (3.2) and (3.3)) would be in them:

$$\Lambda^{(1)} = \frac{1-r}{\bar{l}^2}\left\{\left[(1-r)\left(\bar{c}_1 + m\bar{c}_2\right) - 2\bar{l}\right] - m^0\bar{c}_2\left(i\omega\tau^*_{\sigma 1}\right)\left[1 - r + \left[1 - r - 2\left(\gamma_1 - 1\right)\bar{l}\right]\frac{\tau^*_{\sigma 2}}{\tau^*_{\sigma 1}}\right]\right\},$$

$$\Lambda^{(2)} = 1 - \left(i\omega\tau^*_{T2}\right)\left[1 + \frac{\tau_\sigma}{\tau^*_{\sigma 2}}\right],$$

$$\Lambda^{(3)} = -\,m^0\left(i\omega\tau^*_{\sigma 1}\right)\left\{\left[1 + \frac{\tau_\sigma}{\tau^*_{\sigma 1}}\right] - \left(i\omega\tau^*_{T2}\right)\left[1 + \frac{\tau_\sigma}{\tau^*_{\sigma 2}}\left[1 + \frac{\tau^*_{\sigma 2}}{\tau^*_{\sigma 1}}\right]\right]\right\}.$$

In conformity with the estimate (3.2) $\left|\tau^*_{\sigma 2}/\tau^*_{\sigma 1}\right| \ll 1$ for any vibrations frequency, hence the terms containing this ratio in $\Lambda^{(1)}$ and $\Lambda^{(3)}$ can be neglected as a rule, as compared with one. The influence of nonuniformity of the phase transitions starts to be manifest when the ratio $\tau_\sigma/\tau^*_{\sigma 2}$ in $\Lambda^{(2)}$ and $\Lambda^{(3)}$ that grows with the increase in the frequency ω becomes commensurate with one ($\left|\tau_\sigma/\tau^*_{\sigma 2}\right| \sim 1$) in absolute value.

The expressions for the equilibrium C_e and frozen C_f sound speeds in the steam–drop mixture that are obtained during passage to the limit $\omega \to \infty$ and $\omega \to 0$ can be written in the form

$$C_e = C_1\left[\frac{(1+mr)^2\gamma_e}{(1+m)\,\gamma_1}\right]^{1/2},\ C_f = C_1\left[(1+mr)\frac{2 + r\,(1+3mr)}{2 + r\,(1+3m)}\right]^{1/2}, \qquad (4.2)$$

$$\gamma_e = \left\{1 + \frac{1-r}{\gamma_1\bar{l}^2}\left[(1-r)\left(\bar{c}_1 + m\bar{c}_2\right) - 2\bar{l}\right]\right\}^{-1},$$

where γ_e is the analog of the equilibrium adiabatic index of a two-phase mixture with phase transitions. The frozen velocity C_f is realized in practice ($C(\omega) \approx C_f$) for high ($\omega\tau_V$, $\omega\tau_T \gg 1$) but nevertheless allowable frequencies for this theory $\omega\tau_C \ll 1$.

The dispersion relationship corresponding to the particular case of no phase transitions can be obtained from (4.1) upon passing to the limit $\tau_\sigma \to \infty$. In this case $V(\omega)$ remains the same as in (4.1), while $\theta(\omega)$ has a simpler form with a complex relaxation form τ^*_T determined in (3.4):

$$\Theta(\omega)=\frac{\gamma_1(1+m)\,c_V/c_1-i\omega\tau_T^*}{\gamma_e(1+m)\,c_V/c_1-i\omega\tau_T^*}, \tag{4.3}$$

$$c_V=\frac{c_1-R_1+mc_2}{1+m},\quad c_p=\frac{c_1+mc_2}{1+m},\quad \gamma_e=\frac{c_p}{c_V}.$$

Here c_V and c_p are the equilibrium specific heats for constant volume and pressure, γ_e is the equilibrium adiabatic index of a two-phase mixture without phase transformations. The expressions for the equilibrium and frozen speeds of sound that are obtained from this dependence during passage to the limits $\omega \to 0$ and $\omega \to \infty$ are in agreement with the corresponding expressions (4.2) but with the value of γ_e from (4.3). In the frequency $0 < \omega < 10^{-2}/\tau_{\mu 1}$ considered in [2], the dispersion dependences (4.1) and (4.3) agree with the dependences in [2].

5. Analysis of the Dependences of the Perturbation Propagation Velocities and Damping Factor on the Vibrations Frequency

The characteristic form of the dependences of the phase C_p and group C_g propagation velocities of weak perturbations as well as of their linear damping vector k_{**} on the frequency of vibration ω is shown in Figs. 1-3. The dependences correspond to a steam-water mixture of drop structure ($\alpha_2 < 0.05$) with initial pressure p = 1.0 MPa and are constructed in a frequency band satisfying the requirement of acoustic homogeneity of the medium ($k^{-1} \gg a$). Different series of curves in each figure refer to different mass contents of the condensed phase (numbers near the curves). Different curves in each series are constructed for different values of the accommodation factor β. The solid lines correspond to the value $\beta = 0.04$ ordinarily taken for water, to which $\tau_\sigma/\tau_V = 6\cdot10^{-6}$ corresponds for p = 1.0 MPa, $T = T_S(p)$ and a = 30 μm; the remaining curves illustrate the degree of influence of β on the perturbation dispersion and damping: dash-dot lines are frozen mass transfer ($\beta = 0$); dashes ($\tau_\sigma = \infty$) are quasiequilibrium mass transfer for ($\beta = \infty$); the curves corresponding to finite values of $T_\sigma = T_S$ ($\tau_\sigma = 0$) are within the domain bounded by the limit curves $\beta = 0$ and $\beta = \infty$, and tend to it for high and low frequencies, respectively (dotted line is $\beta = 4\cdot10^{-4}$).

The solid curves are practically in agreement with the dashed curves when $\tau_\sigma \ll |\tau_{\sigma 2}^*|$ and precisely to the frequencies $|\eta_2| \sim 1$, $(\omega\tau_{\lambda 2}) \leqslant 10$, which corresponds to the values $\omega\tau_v \leqslant 10$. The utilization of an assumption about the quasiequilibrium of the mass transfer ($\beta = \infty$) for high frequencies reduces the phase propagation velocity of small perturbations and elevates their linear damping factor. Freezing the mass transfer results in an increase in the velocity and a diminution in the linear damping of the perturbations. The functions $C_g(\omega)$ have a tendency to form local extremums for definite vibrations frequencies (dependent on the drop content in the mixture).

Taking account of the nonstationary effects of interphasal interaction reduces to taking account of the differences between τ_V^*, τ_{T1}^* and τ_{T2}^* and their stationary analogs, which holds for high-frequency perturbations when thinner boundary layers are realized around the drops than quasistationary theory yields. Because of this the interphasal transfer processes proceed more rapidly or in more "equilibrium" than by the quasistationary relationships, moreover with phase shifts with respect to the corresponding thermodynamic forces. Consequently, taking account of the nonstationary results in the dropping of the perturbation propagation velocity, while the linear damping factor increases. The influence of nonstationary effects on the dependence of the phase and group propagation velocities of weak perturbations and their linear damping factor on the vibrations frequency is displayed in Figs. 4-6. The solid lines are constructed taking into account, and the dashed lines without taking into account, the nonstationary effects. The mixture parameters are the same as before, and the accommodation factor is $\beta = 0.04$. The influence of the nonstationary effects (mainly because of the velocity effects [4]) is felt at the frequencies $\omega\tau_{\mu 1} = 9/2(\rho_1^0/\rho_2^0)\omega\tau_v > 10^{-2}$, when a difference between τ_V^* and τ_V starts to appear.

Let us note the following interesting circumstance. If the nonstationary effects are not taken into account, then theory yields such group velocity values $C_g(\omega)$ as can exceed the frozen speed of sound in the mixture $C_f \approx C_1$. The magnitude of the linear damping factor here $k_{**}(\omega) \to \text{const}$ as $\omega \to \infty$. When nonstationary effects are taken into account $C_g(\omega) < C_f$

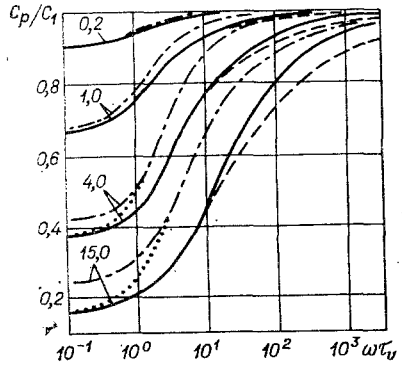

Fig. 1

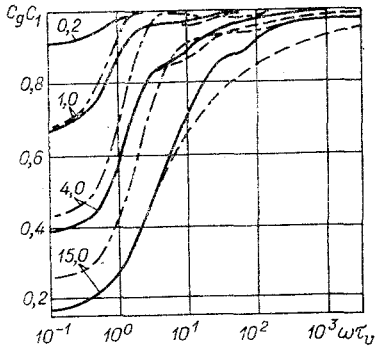

Fig. 2

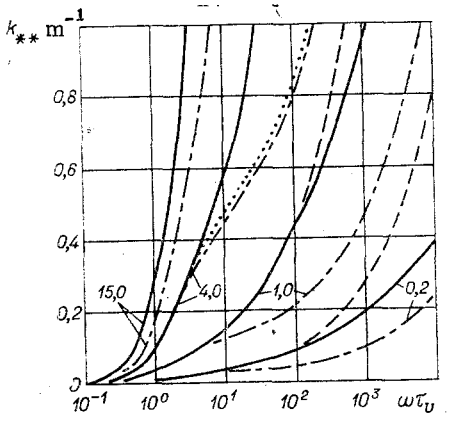

Fig. 3

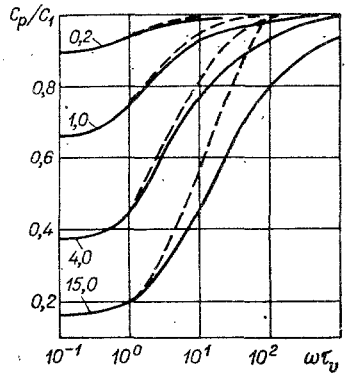

Fig. 4

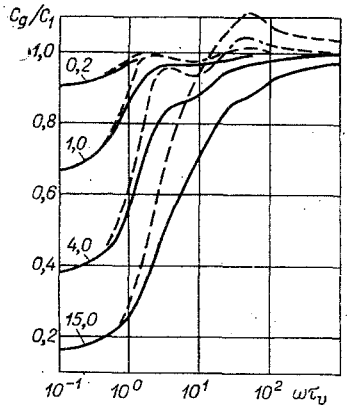

Fig. 5

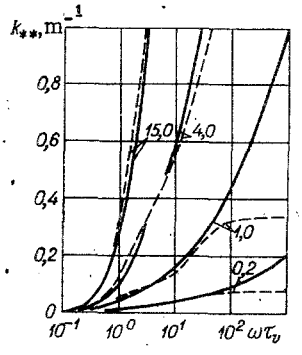

Fig. 6

always, as $\omega \to \infty$ the quantity $k_{**}(\omega) \to \infty$ as the square root of ω. The principal term in the asymptotic of $k_{**}(\omega)$ as $\omega \to \infty$ has the form (remainder term is bounded)

$$k_{**}(\omega) \sim \frac{1}{2} m \sqrt{r} \frac{C_f}{C_1} \frac{\sqrt{\omega\tau_v}}{\tau_v C_1} \left\{ 6\alpha_1 \left[\frac{1-r}{2+r(1+3mr)} \right]^2 + (\gamma_1 - 1)\,\mathrm{Pr}^{-1/2} \left[1 + r \frac{c_1}{c_2} \sqrt{\frac{\varkappa_1}{\varkappa_2}} \right]^{-1} \right\}.$$

For α_2, $r \ll 1$ this formula is simplified

$$k_{**}(\omega) \sim \frac{1}{2} m \sqrt{r_i} \frac{1}{\tau_v C_1} \left\{ \frac{3}{2} + (\gamma_1 - 1)\,\mathrm{Pr}^{-1/2} \right\} \sqrt{\omega\tau_v}.$$

The quantities ω and a are in the complex functions $V(\omega)$ and $\theta(\omega)$ that govern k_*/ω in only the form of the dimensionless combinations $\omega\tau_{\mu 1}$, $\omega\tau_{\lambda 1}$, $\omega\tau_{\lambda 2}$, $\omega\tau_\sigma$. We shall call dimensionless combinations of this kind the frequency-structural parameters. They characterize the ratios between the different relaxation time ($\tau_{\mu 1}$, $\tau_{\lambda 1}$, $\tau_{\lambda 2} \sim a^2$, $\tau_\sigma \sim a$) and the period of the vibrations. Let us set the gas properties (γ_1, C_1, l, ρ_j^0, λ_j, c_j, $j = 1, 2$) and let us extract the

two limit cases when phase transitions are either absent ($\tau_\sigma = \infty$) or occur in quasiequilibrium, i.e., equilibrium on the interphasal boundary ($\tau_\sigma = 0$). In both cases the combination $\omega\tau_\sigma$ is not in the dispersion dependence while the remaining frequency-structural parameters are distinct from each other only by constant factors. This means that in the limit cases in τ_σ mentioned, the dispersion dependence of k_*/ω on ω and a is a function of just the one complex ωa^2, i.e., actually, of just one frequency-structural parameter, for instance, $\omega = \omega\tau_v$ which is the most indicative for gas suspensions. Therefore, for $\omega\tau_\sigma >> 1$ or $\omega\tau_\sigma << 1$ we have

$$C(\omega, a) = C(\omega a^2) = \bar{C}(\bar{\omega}) \quad k_{**}(\omega, a) = \frac{1}{a^2}\bar{k}_{**}(\bar{\omega}), \bar{\omega} = \omega\tau_v = \frac{2\rho_2^v}{9\mu_1}\omega a^2.$$

In other words, for frozen phase transitions ($\beta = 0$) or their quasiequilibrium progress ($\beta = \omega$) the dependences $C_p(\bar{\omega})$ and $C_g(\bar{\omega})$ (the dashed and dash-dot lines in Figs. 1 and 2) are suitable for arbitrary particle sizes from the domain of allowable sizes ($\omega\tau_C << 1$). The non-self-similarity of $C_p(\bar{\omega})$ and $C_g(\bar{\omega})$, i.e., the additional influence of the particle dimension a on the last dependences is manifest because of the temperature nonequilibrium of the interphasal boundaries only in the presence of phase transitions when the difference is felt between T_σ and the saturation temperature T_S. This latter does not hold for $\tau_\sigma/|\tau^*_{\sigma 2}| \sim 1$. .

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